

Statistical Thinking

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Abstract

Referring the historical contributions of quality gurus, we have to see the implication of their contributions rather than listen their stories. These historical contributions are what Liu Yuan-Zhang, the father of Chinese quality, said: “Three Old Pieces of TQM are the Control Chart, the Sampling Acceptance table, and the Orthogonal Experiments. They all have their own mathematical theory and successful applications, in the 1950s and 1960s; most of quality practitioners could understand their usage. When I came back China from U.S. and Japan in 1955, I just brought with these three pieces.” I always put Liu Yuan-Zhang’s saying in my mind, because these “Three Old Pieces” are the basic capabilities why I could do something in the early stage of my quality career, so their theoretical basis and practical application should be understood deeply. On the other hand, solving the quality problems, statistical methods and thinking way is one of the important tools, but also need to integrate the methodologies of essence of substance, process of business and psychology. After years of professional integration in different fields, the quality management gradually formed a multi-value organization team work mode of continuous improvement, leadership, and management by objectives, full participation, common language, problem solving, and response to change. It is the integration of statistical science, management science, engineering science, system science, information science, psychology, etc., in order to improve the quality of human life. Therefore, to understand these philosophies, systems, methodologies, techniques and tools of quality management, we must extensively learn the knowledge of different fields; we must also thoroughly understand the technical methods for various purposes and needs.

Keywords: Control chart; Sampling Acceptance; Orthogonal experiments; TQM

Introduction

The quality management was original from applying the principle of statistics to establish the control charts and acceptance sampling tables, and to provide on-site personnel who do not understand the theory so much, to judge the stability of process and the acceptance of inspection lot. This kind of quality control tools are based on statistical theory, if it does not associate with the practical work, it will lose the value of applications. Speaking of the Control Chart, we cannot but mention the masterpiece of Shewhart: The Economic Control of Quality of Manufactured Product,” just as following description by ASQ website [1]. When W.A. Shewhart (the father of modern quality control) described his books “an indication of direction in which future developments may be expected to take place,” could he have foreseen its enormous impact? This monumental work laid the

foundation discipline, and it remains current today as ever. It began as an attempt to develop a scientific basis for attaining economic control of quality through the establishment of control limits to indicate when quality is varying more than is economically desirable. In his search for better knowledge of economic in manufacture, Shewhart touches upon all aspects of statistical quality control. The book includes a presentation of fundamental concepts and advantages of statistical control; ways of expressing quality of product (a section containing that has been described as on the meaning); the basis for specification of quality control; sampling fluctuations in quality; allowable variability in quality (which contains the first fully developed use of control charts); and quality control in practice. This is required reading for every serious in study of quality control. Sampling acceptance is based on a sampling process to judge the inspection lots from supplier or production line accept or not? How to judge

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to be reasonably, this is the basic problem of sampling acceptance. In other words, what is the sampling plan; Sample size n ? Acceptance number Ac ? The design of sampling plans according to the different risk of protection; different disposal of inspection lot; the use of different occasions, it divided into the standard sampling, rectify sampling, adjustment and continuous production. These methodologies are developed on the basis of mathematical theory. Such kind of methods of judging the truth through random sampling will inevitably lead to the risk of misjudgement. The reason why statistical methods are widely used in all fields of science and has become universally recognized is that it can be calculated the probability of misjudgement, allowing the user to assess the risk of misjudgement. In industry often uses the producer risk α and the consumer risk β to ensure the quality of the judgement of truth. This way of expressing risk by probability is less understandable to the most people, so it is important to cultivate our own statistical thinking way, and is the knowledge that modern citizens should have.

Probability Thinking

Lottery is from 01, 02, 03 ... 47, 48, and 49 choose six numbers randomly, according to the guesser who guesses how many numbers correctly, and wins the prize. Excel hyper geometric distribution HYPGEOMDIST () function is calculated to give the following probability:

The probability of guessing all 6 numbers correctly= $C_6^6 C_0^{43} / C_6^{49} = 0.00000007$: about one ten millionth;

The probability of guessing 5 numbers correctly= $C_5^6 C_1^{43} / C_6^{49} = 0.000018$: about one fifty thousandth;

The probability of guessing 4 numbers correctly= $C_4^6 C_2^{43} / C_6^{49} = 0.0009$: about one thousandth;

The probability of guessing 3 numbers correctly= $C_3^6 C_3^{43} / C_6^{49} = 0.018$: about two hundredth;

The probability of guessing 2 numbers correctly= $C_2^6 C_4^{43} / C_6^{49} = 0.13$: no prize;

The probability of guessing 1 numbers correctly= $C_1^6 C_5^{43} / C_6^{49} = 0.4130$: no prize;

The probability of guessing nil numbers correctly= $C_0^6 C_6^{43} / C_6^{49}$: no prize.

From the above probability calculation, the probability of winning is 0.0186, and the probability of no prize is 0.9814, that is to say, 9,814 notes per 10,000 notes are no prize. The chance of a jackpot is 0.00000007; it is more likely occurred at least 10 million notes per issue. This is why the jackpot often accumulates many issues. The probability of an event occurring must be met by a large number of observations in accordance with theoretical calculations; this is so called the law of large numbers. For example, new products have no defective products in the pilot stage, but there are frequent defective products during mass production, because only one or two hundred pieces are made during the pilot stage, and thousands or tens thousands of pieces are made daily during the mass production. This is also the reason why C_p / C_{pk} can be used to estimate the percent of defective during the mass production (Table 1).

Table 1: C_p / C_{pk} and percent of defective.

Sigma Level	C_p	$k=C_a$	CPU	CPL	C_{pk}	percent of defective (ppm)
1.0	0.333	1.500	-0.167	0.833	-0.167	697,672.1
2.0	0.667	0.750	0.167	1.167	0.167	308,770.2
3.0	1.000	0.500	0.500	1.500	0.500	66,810.6
4.0	1.333	0.375	0.833	1.833	0.833	6,209.7
5.0	1.667	0.300	1.167	2.167	1.167	232.6
6.0	2.000	0.250	1.500	2.500	1.500	3.4

Table 2: Accuracy and Precision.

Quality Indicators	Specification	Distribution Parameter	Statistics	Process Capability	Control Chart	MSA	Statistical Inference
Accuracy	Target: m	μ	$\bar{X}$	C_a	$\bar{X}$	Biased	Unbiasedness
Precision	Tolerance: Δ	σ	s / R	C_p	s / R	%GRR	Efficiency

Probability Distribution Thinking

Every production, service, or management process contains a certain amount of variation due to the presence of many kinds of

causes. Engineers and technicians usually check whether the products are conformed the specifications to ensure that the individual products meet the requirements of the customers. Repeated sampling from the same process in different intervals in

term of time (for example by hour), to observe quality of products or services by measuring its specific characteristics; we will get a lot of data, after organizing the data we will get a distribution. For example, when we are sampling from a standard normal distribution $N(0, 12)$, with sample size $n=10, 100, 1,000, 10,000, 50,000$, we will see it gradually form a distribution. At the same time we also can see the number of nonconformed products occurred when sample size is larger, if the specification is 0 ± 3 (Figure 1,2).

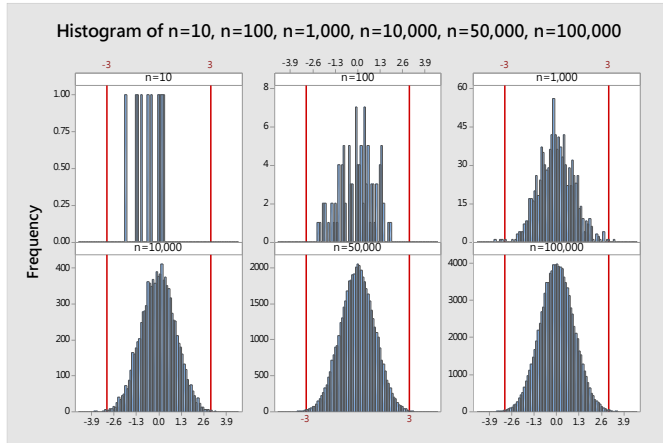


Figure 1: Distribution of characteristics.

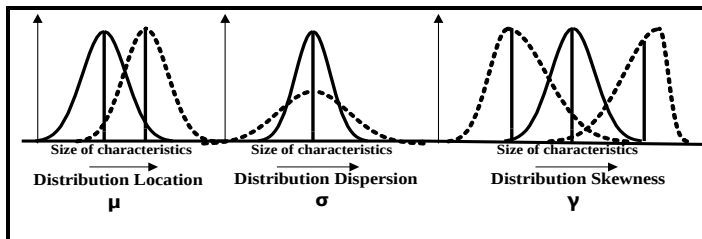


Figure 2: Shewhart's ways of expressing quality.

Shewhart's ways of expressing quality of product by distribution statistically, as shown in Figure 2; the distribution of a characteristic is depicted by the horizontal axis is the size of the characteristics; the vertical axis is the possibility of occurrence of the characteristic. Control charts grasp the variation of process by the location, dispersion and skewness of distribution. In terms of product / process quality characteristics, engineers and technicians usually check the tolerance of the process / product with the specification tolerance to ensure that individual products meet the customer's requirements. If the specification is $m \pm \Delta$, m : target value, Δ : tolerance. $USL = m + \Delta$: upper specification limit; $LSL = m - \Delta$: lower specification limit. If $LSL < Y < USL$, the product is conformed; if $Y < LSL$ or $Y > USL$, the product is nonconformed, as shown in Figure 3. For example, the specification of a steel pipe diameter is 30 ± 0.1 mm, $m = 30$ mm, $\Delta = 0.1$ mm, $USL = 30.1$ mm, and $LSL = 29.9$ mm. On the right side of Figure 3, the GO-

NOGO gauge is checked. If $Y < 30.1$ then GO and $Y > 29.9$ then NOGO, indicating that the product is conformed (Figure 3).

When the process is in the state of under control, the distributions of process over time are fixed. Repeated sampling in different times can be regarded as repeated sampling in the same distribution. What is the sampling distribution of the sample statistics? As shown in Figure 4: A total of k subgroups are sampled, the size of each subgroup is n (Figure 4).

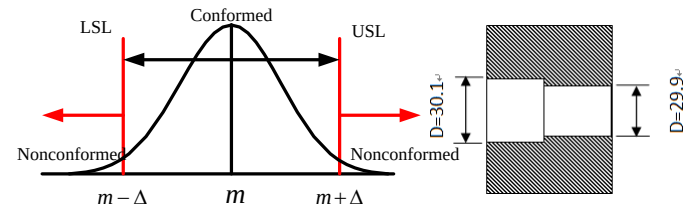


Figure 3: Product characteristics and specification.

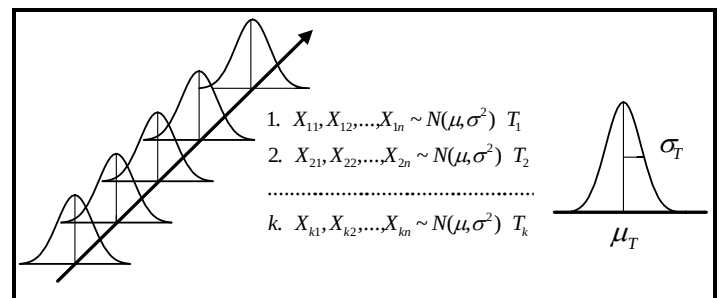


Figure 4: Sampling distribution of statistics.

The control charts are plotted with the statistics, such as, and other statistics, as shown in Figure 5. If there are no special causes for the process, there is only common cause, that is, the distribution of the process is fixed. Repeated sampling in different times can be regarded as repeated sampling in the same distribution. Therefore, we can study the sampling distribution of statistics under the normal distribution to understand the variation pattern of the statistics when repeated sampling from the process with only common cause (Figure 5).

The masterpiece of W.A. Shewhart, in Part IV "Sampling Fluctuation in Quality" used four chapters to explain the sampling distribution of various statistics [2,3]. At the same time, many complicated experiments were carried out to verify the correctness of the theory. We have also been experimented by computer software simulation since our quality control teaching career. These experiments are quite helpful for the establishment of statistical thinking. Figure 6 is the sampling distribution of average ($\bar{X}$), standard deviation (s), and range (R) under normal distribution (Figure 6).

Accuracy and Precision Thinking

However, the control chart is not controlling a product / process characteristic directly. It is mainly to analyze or control process

variations at different times, different lots, different machines, and so on. The variation of the process is not to judge the difference by the result of a test or inspection, but to judge the variation by repeating the observation data of the input / output from the same process.

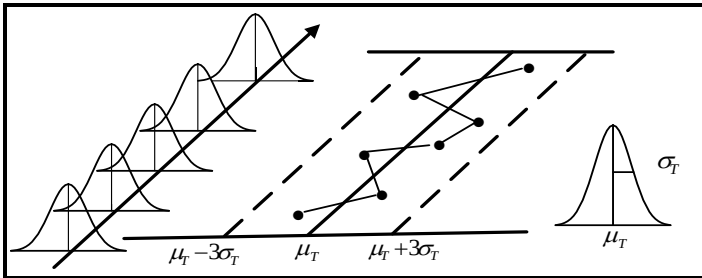


Figure 5: Statistics and control charts.

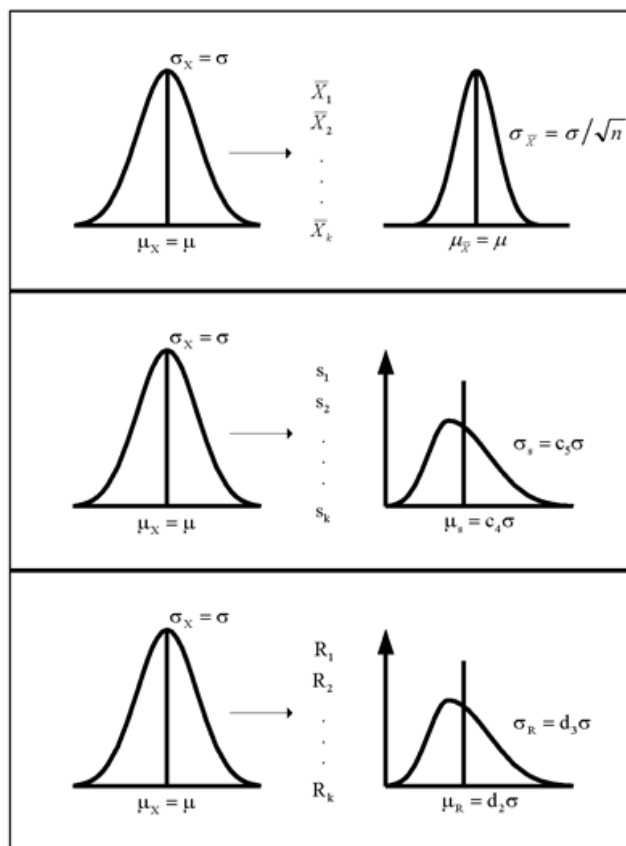


Figure 6: Sampling distribution of $\bar{X}$, S , and R under normal distribution.

There are two ways to judge variation, one is to determine the difference between the mean value μ of the repeated observation data and the target value $m=|\mu-m|$, the smaller the deviation, the more accurate; the other is to compare the standard deviation σ of the observation data with the tolerance Δ ($\sigma=\Delta/z$) or ($\Delta=z\sigma$), the smaller σ is, the more precise is. The product / process quality index is nothing more than the measure of accuracy and precision. The following definition can tell that various quality indicators are concerned with accuracy and precision (Table 2).

Accuracy: manufacturing same product repeatedly, the difference between the mean value μ and the target value m ;

Precision: manufacturing same product repeatedly, the consistence between the products, in other word, the standard deviation σ .

Testing Statistical Hypothesis Thinking

When reading a statistical analysis report, most people often have only superficial concept of p-value and significance. However, in most statistical analysis reports, these methods are used to interpret the meaning of the data. If you do not understand these statistical concepts, you may mislead the analysis report and make a wrong decision. First, define the so-called statistical hypothesis. In order to judge whether accept or reject the hypothesis, the statistician divides the statistical hypothesis into the Null Hypothesis, which is labeled as H_0 ; the Alternative Hypothesis, which is labelled as H_1 . In practice, we often set the null hypothesis H_0 to the factual state suspected by the position of the verifier, which is the same as the principle of presumption of innocence of the judiciary [2]. In terms of problem orientation, there are many problems need to verify, such as, countless goods transactions happening in the industry every day; the science and technology personnel of the science and technology laboratory often have some ideas that dare not confirm their benefits; the production process needs to monitor whether the quality is stable every day; how producers guarantee quality before users use the products. If these problems can be judged by intuition, we probably won't use statistical data to judge, and the null hypothesis H_0 is the truth that we can't judge by intuition. Therefore, we set the null hypothesis of the goods transaction as if the transaction lot is a good lot, the hypothesis of the scientific and technological personnel's creativity is set to be the same as the benefit of the control group, and the null hypothesis of monitoring the process quality is set to the stability of the process quality, and null hypothesis of guaranteeing the product quality is set to quality as standard. However, if these hypotheses describing the truth are not quantified in statistical terms, we still cannot compare them with the data, so the null hypothesis should be expressed in a statistical language that is consistent with the probability distribution calculation; so that the testing statistical hypothesis is carry out. For example, the facts as described above are expressed in statistical language. The so-called alternative hypothesis is a statistical hypothesis that is contrary to the null hypothesis (Table 3).

In practice, the test criteria are used to judge statistical hypotheses; it is often determined whether the H_0 is rejected by the degree of difference between H_0 and the test criteria. The larger the difference, the more H_0 should be rejected. As for how the degree of difference between H_0 and the test criteria is calculated, we define the so-called significance level, and the

probability that the test criteria is present under the H_0 hypothesis, it is the so-called p-value. In general, the smaller the chance, the less we believe that H_0 is true. This is the same thinking way about the truth of our human beings. We are usually observing the occurrence of some events, subjectively, there will be a subjective spectrum in our mind, and this spectrum is the null hypothesis H_0 . After data collection and analysis, if the results of analysis are very different from our subjective spectrum, we often say that it is too unreasonable! This method of judging the truth based on factual data is the testing statistical hypothesis. Part of the research work of statisticians is to find the probability distribution of the test criteria and its inferences in various fields, and to derive the sampling distribution to calculate its accuracy,

precision and p-value. In general, p-value is small, the difference is very significant, statisticians recommend p-value <0.05 or 0.01 , and the most industry standards are also recommended. The implication is that if the same random sampling is repeated 100 times under the null hypothesis H_0 , the occurrence of test criteria is only 5 times or 1 time. And we only have one such random sampling, which is rare under the null hypothesis H_0 , so H_0 should be rejected. The following is an example of some rules for testing special causes of the control chart. Assume that the distribution of process characteristics is normal and the process is under control, the following are some rules for testing of special causes of control charts.

Table 3: Null hypothesis.

H_0 : Factual state	H_0 : Statistical language
H_0 : Transaction lot is a good lot	H_0 : The percent of defective of Transaction lot $p \leq AQL$
H_0 : The benefit of experimental group and control group are the same	H_0 : means of experimental group and control group are equal $\mu_1 = \mu_2$
H_0 : Process is stable	H_0 : Process mean μ and standare deviation σ meet requirements
H_0 : Quality of product meet requirement	H_0 : Product MTBF $\geq 10,000$ hours

The producer's risk $\alpha = P$ [the scatter of points on the control chart | process is under control]. If the probability of the scatter of points occurring is smaller than 0.5%, then we can regard it as special causes. Take -chart as an example, between the upper limit and center line, also lower limit and center lines are equally divided into three zones, namely A, B, and C, that is, zone A is between 2 and 3 ; zone B is between 1 and 2 ; zone C is between 0 and 1 . With the assumption of normal distribution, the probability of occurrence of point drops in each zone can be calculated. For example, the probability of a point drops above or below the center line (outside of the C or in C zone) is 0.5; the point drops outside of 1 (outside of the B or in B zone) is 0.1587; the point drops outside 2 (outside of the A or in A zone) is 0.0228; the point drops outside 3 (outside the control limit) is 0.00135, shown as Figure 7. So, we can use testing statistical hypothesis principle to detect the existence of special cause for the process (Figure 7).

The following are the 8 rules for Testing Special Causes:

- 1 point in or outside A zone, p-value= $0.00135 \times 2 = 0.0027$;
- 9 points in a row in or outside C zone, p-value= $2 \times (0.5)^9 = 0.0039$;
- 6 points in a row, all increasing or all decreasing, p-value= $2 \times (1/6!) = 0.0028$;
- 14 points in a row, alternating up and down; p-value= $2 \times (0.5)^{13} = 0.00024$;
- 2 out 3 points in or outside A zone (same side), p-value= $2 \times (3C2(0.0228)^2 \times (0.9772) + 3C3(0.0228)^3) = 0.0031$;

- 4 out 5 points in or outside B zone (same side), p-value= $2 \times (5C4(0.1587)^4 \times (0.8413) + 5C5(0.1587)^5) = 0.0055$
- 15 points in a row, within C zone (either side), p-value= $(0.6826)^{15} = 0.0033$;
- 8 points in a row, outside C zone (either side), p-value= $(0.3174)^8 = 0.0001$.

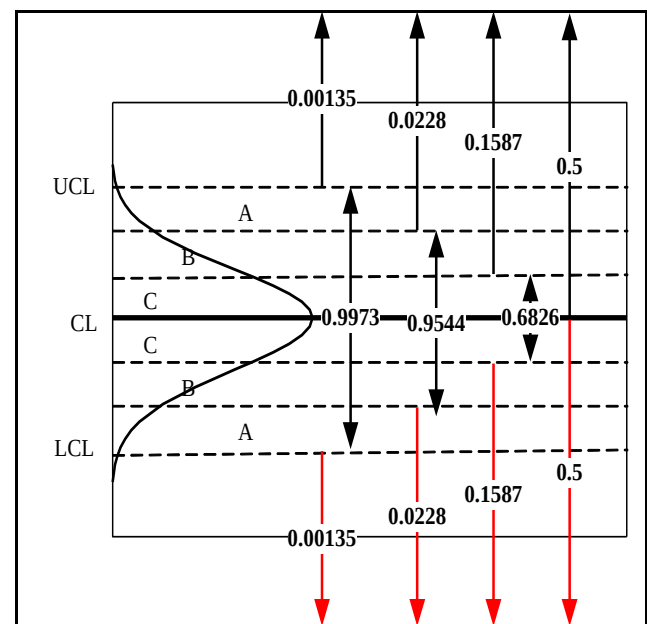


Figure 7: Zone chart.

Uncertain Events Thinking

In formal scientific reports or risk management, there are many uncertain events that need to be assessed as to whether the evidence is possible, credible, and consistent, such as COVID-19 traceability surveys, public opinion surveys, greenhouse gas emissions impact assessments etc. These assessments also require scientific observation, questionnaires, expert interviews, scenario analysis etc., such kind of subjective and objective or quantitative and qualitative analysis. The following summary terms are used to describe the available evidence: limited, medium, or robust; and for the degree of agreement: low, medium, or high. A level of confidence is expressed using five qualifiers: very low, low, medium, high, and very high. For a given evidence and agreement statement, different confidence levels can be assigned, but increasing levels of evidence and degrees of agreement are correlated with increasing confidence. These uncertain events descriptions are defined below [4].

Uncertainty

A cognitive state of incomplete knowledge that can result from a lack of information or from disagreement about what is known or even knowable. It may have many types of sources, from imprecision in the data to ambiguously defined concepts or terminology, or uncertain projections of human behavior. Uncertainty can therefore be represented by quantitative measures (a probability density function) or by qualitative statements (reflecting the judgment of a team of experts).

Agreement

The degree of agreement is the level of concurrence in the literature on a particular finding as assessed by the authors.

Evidence

Information indicating the degree to which a belief or proposition is true or valid. The degree of evidence reflects the amount, quality, and consistency of scientific / technical information on which the authors are basing their findings.

Confidence

The validity of a finding based on the type, amount, quality, and consistency of evidence (mechanistic understanding, theory, data, models, expert judgment) and on the degree of agreement.

Likelihood

The chance of a specific outcome occurring, where this might be estimated probabilistically. This is expressed using a standard terminology: virtually certain 99 – 100 % probability, very likely 90 – 100 %, likely 66 – 100 %, about as likely as not 33 – 66 %, unlikely 0 – 33 %, very unlikely 0 – 10%, exceptionally unlikely 0 – 1 %.

Big Data Thinking

Under the guidance of Internet thinking, one is big data thinking, which is written like this: “In the era of big data, corporate strategy will shift from business mobilization to data mobilization. The data information of massive user access behavior is chaotic, but behind it is the inevitable logic of consumer behaviour. Big data analysis can learn about the inventory and pre-sales of products in various regions, time periods, and consumer groups, and then conduct market judgments, and adjust products and operations based on this.” “Users generally generate data, behavior, and relationship data on the network. The precipitation of these data helps enterprises to make predictions and decisions. In the era of big data, data has become an important asset of enterprises, even core assets.” The value of big data is not only big, but the ability to mine and predict. The core of big data thinking is to understand the value of data, create business value through data processing, data assets become core competitiveness, and small enterprises must have big data also [5]. Traditional quality data is nothing more than variable data, attribute data, defects, internal failure costs, external failure costs, etc., and these data are also collected through data gathering, data processing, statistical analysis, finding root cause, and so on. In the past, quality practitioners also relied on these so-called professional jobs for a position in the company. When these quality data are collected, organized, analysed and monitored automatically by computer, it will be difficult for quality practitioners to keep up with the times. Precision tool machines are also embedded various IOTs due to the development of Internet and IOT technology, collecting machine operation status, component diagnosis, machine life estimation, consumables monitoring, power consumption monitoring, utilization monitoring, and various attributes data analysis, and so on. Production site data is transmitted to the cloud through the Internet, such kind of data mining and forecasting, will be the future of quality professional field worthy of thinking.

Conclusion

Statistical thinking is very important for modern management and technical personnel. When a manager or engineer is required for reporting related issues, if there is additional objective statistical data to enhance the evidence of the report, then it will be easier to convince readers. As D.J. Finney said that “The purpose of statistical science is to provide an objective basis for the analysis of problems in which the data depart from the laws of exact causality [6]”. It is something significant for us to state the



relationship between quality management and statistical science, rather than say quality management relying on the science of statistics, fields of quality lead to some researches in statistics, for example “Engineering and Industrial Statistics”. As for how to cultivate self-fulfilling statistical thinking ability, besides being familiar with statistical theory, but also being learns some of statistical analysis skills.

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